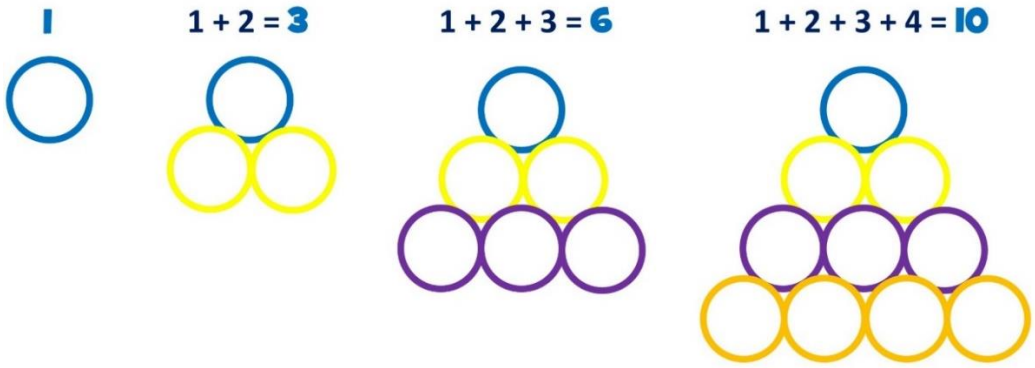
Math Tips: Triangle Numbers

Key Terms:

- **Triangle number**: Any number where that total count of items can be stacked in a triangle, with each row containing 1 more item than the one above it:



Triangle Math: A Handy Shortcut

If you stack balls in a triangle shape, you can find the total number of items in a triangle **knowing only the number in the longest row** – and without adding zillions of numbers! Just multiply the number of balls in the longest row (n) by the next largest number (n+1), then cut the answer in half.

Total Number of Balls = n x (n + 1) / 2

The Math behind the Mystery

Why does the shortcut work? There's a **number** reason and a **shape** reason. To see the **number reason**, watch what happens when you line up a whole bunch of numbers to add:

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10

Notice that **1** + **10** = 11, and **2** + **9** = 11, then **3** + **8**…you're just adding lots of 11s! That pair value will always be (n + 1), and the number of pairs is half of n, since you're carving the numbers into pairs.

Then there's the **shape reason**: That triangle is <u>half</u> of a parallelogram, or tilted rectangle, of height n and width is (n + 1). So its area is n x (n + 1), and then cut in half to get the area of the triangle!